Calculus

**Conceptual Category: Number and Quantity**
**Domain: The Real Number System**
**Cluster: Extend the properties of exponents to rational exponents**
**Essential Questions: How does understanding the structure of an expression increase the understanding of the problem?**

| Framework Standard | Content / Skills | Resources | Instructional Strategies | Assessments |
|---|---|---|---|---|
| N-RN-1<br>Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | Interchange expressions in radical form and rational exponents to help in the differentiation process. | *Lial, Greenwell, Ritchey*, Calculus with Applications 9th edition: 4.1 – 4.3<br><br>*Finney, Demana, Waits, Kennedy*, Calculus: Graphical, Numerical, and Algebraic 3.3<br><br>APCentral | Cooperative learning groups<br><br>Stations<br><br>Whiteboards | Quiz<br>Dip Sticking<br>WAC |
| N-RN-2<br>Rewrite expressions involving radicals and rational exponents using the properties of exponents. | Interchange expressions in radical form and rational exponents to help in the differentiation process.<br><br>Differentiate functions with rational exponents to find the slope of tangent lines.<br><br>Use properties of exponents to simplify expressions in order to find critical points.<br><br>Interchange expressions in radical form and rational exponents to help in the integration process. | *Lial, Greenwell, Ritchey*, Calculus with Applications 9th edition: 4.1 – 4.3, Ch 5, 7.1-7.2<br><br>*Finney, Demana, Waits, Kennedy*, Calculus: Graphical, Numerical, and Algebraic 3.3, Chapter4, 6.1, 6.2, 7.1<br><br>APCentral | Cooperative learning groups<br><br>Stations<br><br>Investigation | Classroom observations<br>Homework review<br>Worksheets<br>Quizzes<br>Investigations<br>Curve Sketching Activity |

**Conceptual Category: Number and Quantity**
**Domain: Quantities**
**Cluster: Reason quantitatively and use units to solve problems.**
**Essential Questions: What is an appropriate unit of measure? How accurate is a measurement?**

| Framework Standard | Content / Skills | Resources | Instructional Strategies | Assessments |
|---|---|---|---|---|
| N-Q-1, 2<br>Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.<br><br>Define appropriate quantities for the purpose of descriptive modeling. | Apply the correct units of measure to related rates problems, velocity, and acceleration.<br><br>Apply the correct units of measure to integrals of net change. | *Lial, Greenwell, Ritchey*, Calculus with Applications 9th edition: 4.1, 6.5<br><br>*Finney, Demana, Waits, Kennedy*, Calculus: Graphical, Numerical, and Algebraic 3.4, 4.6<br><br>APCentral | Cooperative learning groups<br><br>Powerpoint | Quiz<br>Dip Sticking<br>WAC<br>Related Rates Project<br>Sample AP Free Response questions |
| N-Q-3<br>Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | Apply differentials to approximate a change in value of a function.<br><br>Use Linear Approximation and Newton's Method to estimate the value of a function. | *Lial, Greenwell, Ritchey*, Calculus with Applications 9th edition: 6.6<br><br>*Finney, Demana, Waits, Kennedy*, Calculus: Graphical, Numerical, and Algebraic 4.5<br><br>APCentral | Cooperative learning groups<br><br>Stations<br><br>Investigation | Classroom observations<br>Homework review<br>Worksheets<br>Quizzes<br>Investigations<br>Calculator Activity |

**Conceptual Category: Algebra**
**Domain: Seeing structure in Expressions**
**Cluster: Interpret the structure of expressions.**
**Essential Questions: What are the components of an expression? In what ways can you rewrite an algebraic expression?**

| Framework Standard | Content / Skills | Resources | Instructional Strategies | Assessments |
|---|---|---|---|---|
| A-SSE-1a.<br>Interpret expressions that represent a quantity in terms of its context:<br>Interpret parts of an expression, such as terms, factors, and coefficients.<br><br>A-SSE-1b<br>Interpret expressions that represent a quantity in terms of its context:<br>Interpret complicated expressions by viewing one or more of their parts as a single entity. | Apply appropriate rules to evaluating limits, differentiation, and integration. | *Lial, Greenwell, Ritchey,* Calculus with Applications 9th edition: 3.1, 4.1,7.1<br><br>*Finney, Demana, Waits, Kennedy,* Calculus: Graphical, Numerical, and Algebraic 2.1, 3.3, 5.2,5.3,6.1<br><br>APCentral | Cooperative learning groups<br><br>In class practice | Quiz<br>Dip Sticking<br>WAC<br>Sample AP Free Response questions |
| A-SSE-2<br>Use the structure of an expression to identify ways to rewrite it. | Apply appropriate rules to evaluating limits, differentiation, and integration.<br><br>Rewriting functions in order to evaluate the limit algebraically. | *Lial, Greenwell, Ritchey,* Calculus with Applications 9th edition: 3.1, Chapter4, Chapter 7<br><br>*Finney, Demana, Waits, Kennedy,* Calculus: | Cooperative learning groups<br><br>Stations<br><br>Investigation | Classroom observations<br>Homework review<br>Worksheets<br>Quizzes<br>Investigations<br>Calculator Activity |

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| | Rewriting functions in order to differentiate.<br><br>Rewriting the integrand before integrating.<br><br>Writing differentiated functions in simplest form. | Graphical, Numerical, and Algebraic 2.1,2.2,3.3,5.3,6.1,6.2<br><br>APCentral | | |

**Conceptual Category: Algebra**
**Domain: Seeing structure in Expressions**
**Cluster: Write expressions in equivalent forms to solve problems.**
**Essential Questions: What are the components of an expression? In what ways can you rewrite an algebraic expression?**

| Framework Standard | Content / Skills | Resources | Instructional Strategies | Assessments |
|---|---|---|---|---|
| A-SSE-3a.<br>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression:<br><br>Factor a quadratic expression to reveal the zeros of the function it defines. | Determine where functions are differentiable.<br><br>Determine horizontal asymptotes of a function when sketching curves. | *Lial, Greenwell, Ritchey*, Calculus with Applications 9th edition: 3.5, 5.4<br><br>*Finney, Demana, Waits, Kennedy*, Calculus: Graphical, Numerical, and Algebraic 3.2, 4.3<br><br>APCentral | Cooperative learning groups<br><br>In class practice | Quiz<br>Curve Sketching Activity<br>WAC<br>Sample AP Free Response questions |

**Conceptual Category: Algebra**
**Domain: Arithmetic with Polynomials and Rational Expressions**
**Cluster: Rewrite rational expressions**
**Essential Questions: How can rational functions be rewritten?**

| Framework Standard | Content / Skills | Resources | Instructional Strategies | Assessments |
|---|---|---|---|---|
| A- APR-6, 7<br>Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.<br><br>(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | Apply appropriate rules to evaluating limits, differentiation, and integration.<br><br>Rewriting functions in order to evaluate the limit algebraically. | Lial, Greenwell, Ritchey, Calculus with Applications 9th edition: 3.1, Chapter4, Chapter 7<br><br>Finney, Demana, Waits, Kennedy, Calculus: Graphical, Numerical, and Algebraic 2.1,2.2,3.3,5.3,6.1,6.2<br><br>APCentral | Cooperative learning groups<br><br>Stations<br><br>Investigation<br><br>Think Pair Share | Quiz<br>Dip Sticking<br>WAC<br>Sample AP Free Response questions |

**Conceptual Category: Algebra**
**Domain: Creating Equations**
**Cluster: Create equations that describe numbers or relationships.**
**Essential Questions: How do you model a situation in two variables? What are key words that can be useful when writing an equation from a word problem?**

| Framework Standard | Content / Skills | Resources | Instructional Strategies | Assessments |
|---|---|---|---|---|
| A-CED-2<br>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | Create appropriate equations to model situations in optimization and related rate problems.<br><br>Interpret graphs of modeled functions to ascertain maximum or minimum values in optimization problems. | *Lial, Greenwell, Ritchey*, Calculus with Applications 9th edition: 6.2, 6.5<br><br>*Finney, Demana, Waits, Kennedy*, Calculus: Graphical, Numerical, and Algebraic: 4.4, 4.6<br><br>APCentral | Cooperative learning groups<br><br>Stations<br><br>Investigation | Quiz<br>Dip Sticking<br>WAC<br>Sample AP Free Response questions<br>Related Rates Project |
| A-CED-3<br>Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | Determine the appropriate constraints on variables in an optimization problem. | *Lial, Greenwell, Ritchey*, Calculus with Applications 9th edition: 6.2<br><br>*Finney, Demana, Waits, Kennedy*, Calculus: Graphical, Numerical, and Algebraic: 4.4<br><br>APCentral | Cooperative learning groups<br><br>Stations<br><br>Investigation | Quiz<br>Dip Sticking<br>WAC<br>Sample AP Free Response questions<br>Ticket Out |

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| A-CED-4<br>Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | Rewrite equations that represent constraints in terms of one variable in optimization problems.<br><br>Rewrite the rate of change in terms of one variable in Related Rates problems. | *Lial, Greenwell, Ritchey*, Calculus with Applications 9th edition: 6.2, 6.5<br><br>*Finney, Demana, Waits, Kennedy*, Calculus: Graphical, Numerical, and Algebraic: 4.4, 4.6<br><br>APCentral | Cooperative learning groups<br><br>Stations<br><br>Investigation | Quiz<br>Dip Sticking<br>WAC<br>Sample AP Free Response questions<br>Ticket Out |

**Conceptual Category: Algebra**
**Domain: Reasoning with equations and inequalities**
**Cluster: Represent and solve equations and inequalities graphically.**
**Essential Questions: What does it mean to intersect? What does it mean when two graphs intersect?**

| Framework Standard | Content / Skills | Resources | Instructional Strategies | Assessments |
|---|---|---|---|---|
| A-REI-11<br>Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive | Utilize the points of intersection to find the limits of integration when finding the area between two curves. | Finney, Demana, Waits, Kennedy, Calculus: Graphical, Numerical, and Algebraic 7.2<br><br>APCentral | Graphing Calculator<br><br>Observation<br><br>Investigation<br><br>Group work | Quiz<br>Sample AP Free Response questions |

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| approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | | | | |

**Conceptual Category: Algebra**
**Domain: Interpreting Functions**
**Cluster: Understand the concept of a function and use function notation.**
**Essential Questions: What does inverse mean? What is domain and range?**

| Framework Standard | Content / Skills | Resources | Instructional Strategies | Assessments |
|---|---|---|---|---|
| F-IF-2<br>Use function notation, evaluate functions for inputs in their domains,and interpret statements that use function notation in terms of a context.. | Evaluate a function and its derivative in order to evaluate the derivative of the inverse function. | Finney, Demana, Waits, Kennedy, Calculus: Graphical, Numerical, and Algebraic 3.8<br><br>APCentral | Graphing Calculator<br><br>Observation<br><br>Investigation<br><br>Group work | Quiz<br>Sample AP Free Response questions |

**Conceptual Category: Algebra**
**Domain: Interpreting Functions**
**Cluster: Interpret functions that arise in application in terms of the context.**
**Essential Questions: What does it mean to graph a function? What does maximum or minimum mean?**

| Framework Standard | Content / Skills | Resources | Instructional Strategies | Assessments |
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| F-IF-4<br>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. | Use the derivative to find and graph intervals of increase and decrease and absolute and local extrema.<br><br>Use the second derivative to find and plot inflection points, and to graph intervals of concave up and concave down.<br><br>Interpret key features of a graph given a table of values which include information about the function itself and its derivatives.<br><br>Evaluate limits graphically. | *Lial, Greenwell, Ritchey,* Calculus with Applications 9th edition: 3.1, Chapter 5<br><br>*Finney, Demana, Waits, Kennedy,* Calculus: Graphical, Numerical, and Algebraic: 2.1, 4.1-4.3<br><br>APCentral | Graphing Calculator<br><br>Observation<br><br>Investigation<br><br>Group work<br><br>Whiteboards | Quiz<br>Sample AP Free Response questions<br>Curve Sketching activities |
| F-IF-6<br>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | Relate average rate of change with the limit of the different quotient as *h* approaches zero (the definition of the derivative!!)<br><br>Evaluate the derivative of a function based on a table of values.<br><br>Estimate the slopes of tangent lines to a curve to graph the derivative function. | *Lial, Greenwell, Ritchey,* Calculus with Applications 9th edition: 3.3,3.4,3.5<br><br>*Finney, Demana, Waits, Kennedy,* Calculus: Graphical, Numerical, and Algebraic: 2.4,3.1,3.3<br><br>APCentral | Observation<br><br>Investigation<br><br>Group work<br><br>Whiteboards | Quiz<br><br>Derivative Graph Matching activity<br><br>Ticket Out |

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| F-IF-7c<br>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases:<br><br>Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | Use the derivative to find and graph intervals of increase and decrease and absolute and local extrema.<br><br>Use the second derivative to find and plot inflection points, and to graph intervals of concave up and concave down.<br><br>Use limits to determine end behavior | *Lial, Greenwell, Ritchey*, Calculus with Applications 9th edition: 3.1, Chapter 5<br><br>*Finney, Demana, Waits, Kennedy*, Calculus: Graphical, Numerical, and Algebraic: 2.2, 4.1-4.3<br><br>APCentral | Graphing Calculator<br><br>Observation<br><br>Investigation<br><br>Group work<br><br>Whiteboards | Quiz<br>Sample AP Free Response questions<br>Curve Sketching activities |
| F-IF-7d<br>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases:<br><br>Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | Use the derivative to find and graph intervals of increase and decrease and absolute and local extrema.<br><br>Use the second derivative to find and plot inflection points, and to graph intervals of concave up and concave down.<br><br>Use limits to determine vertical and horizontal asymptotes. | *Lial, Greenwell, Ritchey*, Calculus with Applications 9th edition: 3.1, Chapter 5<br><br>*Finney, Demana, Waits, Kennedy*, Calculus: Graphical, Numerical, and Algebraic: 2.2, 4.1-4.3<br><br>APCentral | Graphing Calculator<br><br>Observation<br><br>Investigation<br><br>Group work<br><br>Whiteboards | Quiz<br>Sample AP Free Response questions<br>Curve Sketching activities |

**Conceptual Category: Algebra**
**Domain: Building functions**
**Cluster: Build new functions from existing functions.**
**Essential Questions: How do you find an inverse function? What does the graph look like when you switch the domain and range of a function?**

| Framework Standard | Content / Skills | Resources | Instructional Strategies | Assessments |
|---|---|---|---|---|
| F-BF-4d<br>Find inverse functions:<br><br>(+) Produce an invertible function from a noninvertible function by restricting the domain. | Prove and utilize the derivative of an inverse trig function. | Finney, Demana, Waits, Kennedy, Calculus: Graphical, Numerical, and Algebraic 3.8<br><br>APCentral | Graphing Calculator<br><br>Observation<br><br>Investigation<br><br>Group work | Quiz<br>Sample AP Free Response questions |
| F-BF-5<br>(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. | Use the relationship between exponential and log functions to prove the derivative functions.<br><br>Use properties of exponential and log functions to solve equations involving the derivative of such functions. | *Lial, Greenwell, Ritchey*, Calculus with Applications 9th edition: 4.4,4.5<br><br>*Finney, Demana, Waits, Kennedy*, Calculus: Graphical, Numerical, and Algebraic: 3.9<br><br>APCentral | Graphing Calculator<br><br>Observation<br><br>Investigation<br><br>Group work | Quiz<br>Sample AP Free Response questions |

**Conceptual Category: Functions**
**Domain: Trigonometric Functions**
**Cluster: Extend the domain of trigonometric functions using the unit circle.**
**Essential Questions: What is trig? What are special right triangles?**

| **Framework Standard** | **Content / Skills** | **Resources** | **Instructional Strategies** | **Assessments** |
|---|---|---|---|---|
| F-TF-3<br>(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. | Evaluate the derivative of trig functions without the use of a calculator.<br><br>Evaluate the definite integral of a trig function without the use of a calculator<br><br>Write the equation of the tangent line to the graph of a trig function. | *Lial, Greenwell, Ritchey,* Calculus with Applications 9th edition: 7.4, 13.2<br><br>*Finney, Demana, Waits, Kennedy,* Calculus: Graphical, Numerical, and Algebraic: 3.5, 5.3, 6.2, 7.1<br><br>APCentral | Cooperative learning groups<br><br>Stations<br><br>Investigation | Quiz<br>Dip Sticking<br>WAC<br>Sample AP Free Response questions |

**Conceptual Category: Geometry**
**Domain: Geometric Measurement and Dimension**
**Cluster: Explain volume formulas and use them to solve problems**
**Essential Questions: What is volume? What is the difference between area and volume?**

| **Framework Standard** | **Content / Skills** | **Resources** | **Instructional Strategies** | **Assessments** |
|---|---|---|---|---|
| G-GMD-3<br><br>Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | Apply area and volume formulas to related rate problems.<br><br>Find the volume of a solid by using the definite integral and area of a cross section.<br><br>Find the volume of a solid of revolution by using the definite integral. | *Lial, Greenwell, Ritchey,* Calculus with Applications 9th edition: 6.5, 8.2<br><br>*Finney, Demana, Waits, Kennedy,* Calculus: Graphical, Numerical, and Algebraic: 4.6, 7.3<br><br>APCentral | Cooperative learning groups<br><br>Stations<br><br>Investigation | Quiz<br>WAC<br>Sample AP Free Response questions<br>Play-Dough Activity<br>Related Rates Activity |

**Conceptual Category: Geometry**
**Domain: Geometric Measurement and Dimension**
**Cluster: Visualize relationships between two dimensional and three dimensional objects.**
**Essential Questions: What is volume? What is the difference between area and volume?**

| Framework Standard | Content / Skills | Resources | Instructional Strategies | Assessments |
|---|---|---|---|---|
| G-GMD-4<br><br>Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two dimensional objects. | Find the volume of a solid by using the definite integral and area of a cross section.<br><br>Find the volume of a solid of revolution by using the definite integral. | *Lial, Greenwell, Ritchey,* Calculus with Applications 9th edition: 8.2<br><br>*Finney, Demana, Waits, Kennedy,* Calculus: Graphical, Numerical, and Algebraic: 7.3<br><br>APCentral | Cooperative learning groups<br><br>Stations<br><br>Investigation | Quiz<br>WAC<br>Sample AP Free Response questions<br>Play-Dough Activity |

**Conceptual Category: Statistics and Probability**
**Domain: Interpreting Categorical and Quantitative Data.**
**Cluster: Interpret linear models.**
**Essential Questions: What is slope? What is rate of change?**

| **Framework Standard** | **Content / Skills** | **Resources** | **Instructional Strategies** | **Assessments** |
|---|---|---|---|---|
| S-ID-7<br><br>Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | Find the slope of a tangent line and interpret its meeting.<br><br>Draw a slope field and use it to graph the solution to the initial value problem. | *Lial, Greenwell, Ritchey,* Calculus with Applications 9th edition: Chapter 4<br><br>*Finney, Demana, Waits, Kennedy,* Calculus: Graphical, Numerical, and Algebraic: 2.4, Chapter 3, 6.1<br><br>APCentral | Cooperative learning groups<br><br>Stations<br><br>Investigation | Quiz<br>WAC<br>Sample AP Free Response questions |